AD-A041 512 TENNESSEE UNIV KNOXVILLE F/G 12/1
ITERATIVE METHODS FOR SINGULAR SYSTEMS OF EQUATIONS.(U)
JUN 77 R J PLEMMONS DAHC04-74-C-0019
UNCLASSIFIED ARO-11755.10-M NL

1 OF 1
AD
A041512

END

DATE
FILMED
7-77

ARO 11755.10-M

ITERATIVE METHODS FOR SINGULAR SYSTEMS OF EQUATIONS

FINAL REPORT

ROBERT J. PLEMMONS

JUNE 3, 1977

U. S. ARMY RESEARCH OFFICE

CONTRACT NO. DAHC04-74-C-0019

UNIVERSITY OF TENNESSEE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER (19) 11755.10-M (18) ARO	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER (9)	
4. TITLE (and Subtitle) (6) ITERATIVE METHODS FOR SINGULAR SYSTEMS OF EQUATIONS,	5. TYPE OF REPORT & PERIOD COVERED Final Report, 15 Apr 74 - 14 Apr 77,
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) Robert J. Plemmons	8. CONTRACT OR GRANT NUMBER(s) (15) DAHC04-74-C-0019
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Tennessee Knoxville, Tennessee 37916 346 650	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709	12. REPORT DATE (11) 3 June 1977
	13. NUMBER OF PAGES 6
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) (12) 6p.	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Approximation	Partial differential equations
Iterations	Markov processes
Equations	Statistical Analysis
Geodesy	Structural properties
Finite difference theory	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The objectives of the research were to study direct and iterative methods for approximating solutions to systems of linear equations Ax=b, where the coefficient matrix A is singular or in general rectangular. Such problems arise in geodesy calculations, in finite difference and finite element methods for approximating solutions to certain types of partial differential equations such as the Neumann problem or Poisson's equation on a sphere, in statistical calculations in structural analysis and in Markov analysis.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

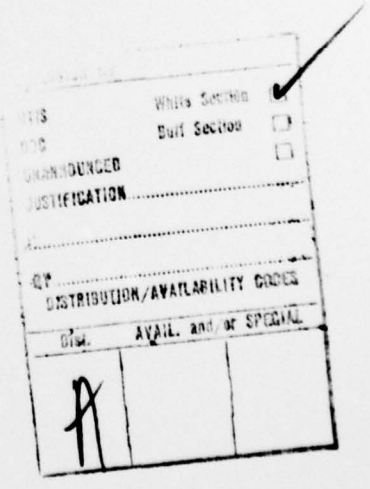
LB Unclassified 346650

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

I. THE PROBLEMS STUDIED

The objectives of the research were to study direct and iterative methods for approximating solutions to systems of linear equations $Ax = b$, where the coefficient matrix A is singular or in general rectangular. Such problems arise in geodesy calculations, in finite difference and finite element methods for approximating solutions to certin types of partial differential equations such as the Neumann problem or Poisson's equation on a sphere, in statisitical calculations, in structural analysis and in Markov analysis.

Direct and iterative methods were to be investigated. Direct algorithms for obtaining minimal ℓ_2 - solutions to consistent rectangular systems of linear equations were to be developed and compared numerically. The primary emphasis of the research was to be placed iterative methods and the case where the coefficient matrix A of the system is large and sparse. Both the consistent and the inconsistent cases were to be investigated here. In the consistent case, special emphasis was to be placed upon certain types of matrices A, such as M-matrices, that commonly arise in these applications. Finally, the inconsistent case was to be studied with the purpose of developing an SOR type iterative method for large sparse linear least squares problems.



II. SUMMARY OF THE MOST IMPORTANT RESULTS

The most important research accomplishments by the principal investigator are as follows:

1. Minimum ℓ_2 - Solutions to Rectangular Systems of Linear Equations. This joint research, with R. E. Cline, involved the development and testing of some new algorithms for using various combinations of orthoginal decomposition methods and elimination methods for computing solutions to $Ax = b$, minimal with respect to the Euclidean norm. Some new techniques were designed, implemented on the computer and were compared with existing techniques. Growing out of this research was a 28 page paper in the SIAM Review.
2. Iterative Methods for Consistent Linear Systems. This research involved the investigations of iterative methods for approximating solutions to consistent systems, based upon various splittings of A into $A = M - N$ with M nonsingular. Particular attention was given to the study of the convergence of such methods and upon extending the usual linear stationary iterative methods, such as the SOR method, to the singular case. Some new convergence results were obtained and five papers were published on this subject, some with joint authors C. Meyer, M. Neumann and J. Ortega.
3. M-matrices and Related Topics. The investigation of iterative methods for consistent linear systems led in a natural way to the study of the properties of some important classes of matrices. Particular attention was given to the roles of matrix monotonicity and nonnegative matrices in the mathematical sciences. Some of the major results of this research were (1) characterizations of various types of matrix monotonicity and their relationships to monotone iterations, (2) a solution of the classical problem of characterizing in a useful way the diagonal stability of a real, square matrix and (3) the collection and classification of

40 characterizations of nonsingular M-matrices. Five papers or reports were written on these subjects, some with joint authors G. Barker, A. Berman and M Neumann.

4. Block SOR Type Methods for Certain Large Sparse Linear Least Squares Problems. One of the primary research contributions under this contract was the investigation of block iterative methods for solving the large sparse linear least squares problem

$$\min_x ||b - Ax||_2$$

where A has full column rank and has been permuted into the form

$$A = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} \quad , A_1 \text{ nonsingular.}$$

It has been shown that a block SOR type iterative method is both feasible and practical whenever

$$A_1^t A_1 - A_2^t A_2$$

is positive definite. The method is especially attractive for solving the linear least squares problem with equality constraints. Methods for approximating the optimum SOR relaxation parameter (which is shown to lie in the interval (.9,1] were obtained, thus extending some earlier work of Chen and Gentleman. A preliminary report on this work has been written with G. Melendez and research is continuing on this project.

The principal investigator feels that the majority of the research objectives of this three year contract have been met and that the overall results will provide useful tools for U.S. Army Researchers and others working in these areas.

No scientific personnel other than the principal investigator was employed on this contract.

III. LIST OF PUBLICATIONS

A. Papers Published or Accepted for Publicaton.

1. "ℓ_2 - solutions to undertermined linear systems", SIAM Review, 18(1976) pp. 92- 106, with R. Cline.
2. "Eight types of matrix monotonicity", Linear Algebra and Its Applications 13(1976), pp. 115-123, with A. Berman.
3. "Regular splittings and the discrete Neumann Problem", Numerische Mathematic, 25(1976), pp. 153-161.
4. "M-matrices leading to semiconvergent splittings", Linear Algebra and Its Applications, 15(1976), pp. 243-252.
5. "Positive diagonal solutions to the Lyapunov equation", to appear in Linear and Multilinear Algebra, 1977, with G. Barker and A. Berman.
6. "M-matrix characterizations I: Nonsingular M-matrices", to appear in Linear Algebra and Its Applications, 1977.
7. "Inverse-positively and splittings of M-matrices", to appear in Linear Algebra and Its Applications, with M. Neumann.
8. "Convergent powers of a matrix with applications to iterative methods for singular linear systems", to appear in the SIAM J. on Numerical Analysis, 14(1977), No. 4, with C. D. Meyer.

B. Technical Reports and Papers Submitted.

1. "Block iterative methods for large sparse linear least squares problems", with G. Melendez.
2. "Convergent powers of nonnegative matrices."
3. "Rectangular M-matrices and applications", with M. Neumann.
4. "Convergent splittings of Hermitian matrices", with James M. Ortega.

In addition, the book Nonnegative Matrices in the Mathematical Sciences, in preparation, with A. Berman, contains some new and unpublished material. The book will be published by Academic Press in 1978.